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XOS SONLAR VA XOS FUNKSIYALAR

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Annotatsiya: *mazkur maqolada xos sonlar va xos funksiyalar, ularga oid amallar haqida izlanishlar va ko'rsatmalar berilgan.*
Kalit so'zlar: *xos sonlar, son, funksiya, tenglama, hosila, funksiya.*

Biz avval differensial tenglamalar nazariyasida xos funksiyalar va xos sonlarning kelib chiqishiga to'xtalib o'tmoqchimiz. Misol uchun, issiqlik o'tkazuvchanlik nazariyasiga oid bo'lgan

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2},\ \left(0<x<l,\ 0<t\le T\right) \qquad (2.\ 1)$$

tenglamani olib ko'ramiz. Tenglamaning yechimi G sohada aniqlanadi (49-rasm). Chegaradagi shartlar

$$u\left(0,\ t\right)=u\left(l,\ t\right)=0 \qquad (2.\ 2)$$

va boshlang'ich shart

$$u\left(x,\ 0\right)=u_0\left(x\right),\ 0\le x\le t. \qquad (2.3)$$

berilgan deb hisoblanadi. Tenglama yechimini (Fure usuli bilan)

$$u\left(x,\ t\right)=X\left(x\right)T(t) \qquad (2.4)$$

ko'rinishda ifoda qilishga urinib ko'ramiz. (2.4) dan

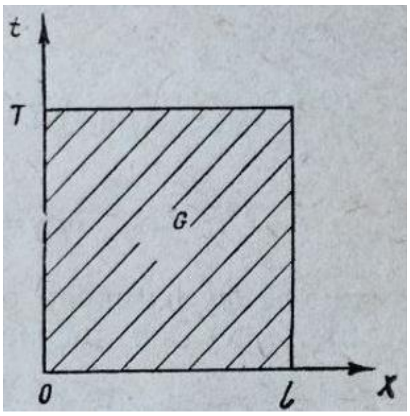
$$\frac{\partial^2 u}{\partial x^2}=X''(x)T(t),$$

$$\frac{\partial u}{\partial t}=X(x)T'(t)$$

kelib chiqadi.

Shuning uchun (2.1) ni quyidagi ko'rinishda yozish mumkin:

$$T'X=TX''.$$



49-shakl

Bundan esa

$$\frac{T'}{T}=\frac{X''}{X}$$

Oxirgi tenglikning chap tomonida faqat t ga, o'ng tomonida esa faqat x ga bog'liq funksiyalarning nisbati turibdi. Demak, ular biror - λ o'zgarmas songa teng bo'lishi kerak:

$$\frac{T'}{T}=\frac{X''}{X}=-\lambda,$$

bu yerda manfiy ishora keying hisoblashlar qulay bo'lishi uchun olindi. Shunday qilib, ikkita

$$X''(x)+\lambda X(x)=0 \qquad (2.5)$$

$$T'(t)+\lambda T(t)=0 \qquad (2.6)$$

tenglamani hosil qildik. (2.5) uchun chegaraviy shart

$$X(0)=0, \quad X(l)=0 \qquad (2.5')$$

bo'ladi. (2. 5) tenglamaning $(2.\ 5')$ chegaraviy shartlarni qanoatlantiruvchi, nolga teng bo'lmagan yechimlariga mos keluvchi λ ning qiymatlari *xos sonlar*, yechimlarning o'zi esa *xos funksiyalar* deb ataladi. (2.5) tenglamaning xos sonlarini λ_k, ularga mos keladigan xos funksiyalarni esa X_k desak, ular

$$\lambda_k=\frac{k^2\pi^2}{l^2}, \quad X_k(x)=\sqrt{\frac{2}{l}}\ \sin\frac{k\pi x}{l}, \quad k=1,2,3,... \qquad (2.7)$$

ga teng bo'ladi.

Xos funksiyalarning funksiyonal qatorlar nazariyasida isbotlanadigan quyidagi uchta xususiyatini eslatib o'tmoqchimiz:

$$1)\ \int_0^1 X_k(x)X_m(x)dx=\delta_{km}, \qquad (2.8)$$

$$\delta_{km}=\begin{cases}0, & k\neq m,\\ 1, & k=m\end{cases}$$

ya'ni xos funksiyalar ortogonal sistema tashkil etadi;

2) $X_k(x)$ dan hosila olib, quyidagicha yozamiz:

$$X_k'(x)=\sqrt{\lambda_k}\sqrt{\frac{2}{l}}\cos\frac{k\pi x}{l}=\sqrt{\lambda_k}\ \overline{X}_k(x) \qquad (2.9)$$

va

$$\int_0^l \overline{X}_k(x)\overline{X}_m(x)dx=\delta_{km}, \qquad (2.10)$$

ya'ni xos funksiyalarning hosilalari ham ortogonal sistema tashkil etadi;

3) agar $f(x)$ ikki marta differensiallanuvchi bo'lsa va $f(0)=f(l)=0$ shartlar bajarilsa, u holda uni xos funksiyalar orqali yaqinlashuvchi qator ko'rinishida quyidagicha tasvirlash mumkin:

$$f(x)=\sum_{k=1}^{\infty} f_k X_k(x) \qquad (2.11)$$

bu yerda

$$f_k = \int_0^l f(x) X_k(x) dx$$

va

$$\| f \| = \int_0^l f^2(x) dx$$

(2.6) tenglamaning λ_n ga oid yechimi

$$T_n = C_n e^{-\lambda_n t}, \ (n = 1,\ 2\ ,\ ...) \tag{2.12}$$

ga teng, bu yerda C_n -o’zgarmas sonlar.

(2.1) tenglama chiziqli bo’lganligidan uning yechimini

$$u(x,t) = \sum_{n=1}^{\infty} X_n T_n = \sum_{n=1}^{\infty} C_n \sqrt{\frac{2}{l}} \sin \frac{k\pi x}{l} e^{-\lambda_n t} \tag{2.13}$$

ko’rinishda yozish mumkin.

C_n ixtiyoriy sonlarning qiymatlari boshlang’ich shartlardan aniqlanadi. Agar (2.13) da $t = 0$ deb olsak,

$$u(x,0) = u_0(x) = \sum_{n=1}^{\infty} C_n \sqrt{\frac{2}{l}} \sin \frac{n\pi x}{l}.$$

(2. 7) va (2. 8) ga asosan

$$C_n = \sqrt{\frac{2}{l}} \int_0^l u_0(x) \sin \frac{n\pi x}{l} dx \tag{2.14}$$

bo’ladi.

Chekli ayirmali masalalarda ham xos son va xos funksiyalar tushunchalarini kiritish uchun (2. 5), (2. $5'$) masalaga mos keladigan va Shturm- Lyuvill masalasi deb ataladigan quyidagi

$$y_{\bar{x}x} + \lambda y = 0, \tag{2.15}$$

$$y_0 = y_N = 0 \tag{2.16}$$

chekli ayirmali masalani olamiz. (2.15) ni indekslar yordamida yozsak,

$$y_{i+1} - 2\left(1 - \frac{h^2\lambda}{2}\right)y_i + y_{i-1} = 0, \quad i = \overline{1, N-1} \tag{2.17}$$

hosil bo’ladi. Buning yechimini $y(x) = \sin\alpha x$ ko’rinishda izlaymiz, bu yerda α - hozircha noma’lum son. U holda

$$y_{i+1} + y_{i-1} = \sin\alpha(x+h) + \sin\alpha(x-h) = 2\sin\alpha x \ \cos\alpha h$$

ni e’tiborga olsak, (2.17) dan

$$2\sin\alpha x \ \cos\alpha h = 2\left(1 - \frac{h^2\lambda}{2}\right)\sin\alpha x$$

yoki biz notrivial yechimlarni izlayotgan bo’lganligimiz, ya’ni $y(x) = \sin\alpha x \neq 0$ bo’lganligi uchun

$$\cos\alpha h = 1 - \frac{h^2\lambda}{2},$$

va nihoyat,

$$\lambda = \frac{4}{h^2}\sin^2\frac{ah}{2}$$

kelib chiqadi. (2.16) ga asosan

$$\sin\alpha l = 0, \tag{2.18}$$

bundan

$$\alpha = \alpha_k = \frac{k\pi}{l}, \ k = \overline{1, \ N-1} \tag{2.18’}$$

hosil bo’ladi. Shunday qilib, (2.15), (2.16) ayirmali masala uchun xos son va xos funksiyalar quyidagicha yoziladi:

$$\lambda_k = \frac{4}{h^2}\sin^2\frac{k\pi h}{2l}, \tag{2.19}$$

$$y^{(k)}(x) = \sin\frac{k\pi x}{l}, \ k = \overline{1, \ N-1} \tag{2.20}$$

Bular uchun quyidagi xossalar o’rinlidir:

1) $$0<\lambda_1=\frac{4}{h^2}\sin^2\frac{\pi h}{2l}<\lambda_2<.....<\lambda_{N-1}=\frac{4}{h^2}\sin^2\frac{\pi h(N-1)}{2l}\le\frac{4}{h^2};$$

2) $y^{(k)}$ va $y^{(m)}$ (2.15), (2.16) masalaning bir-biridan farqli 2 ta xos soniga mos
keluvchi funksiyalar bo’lsin, u holda bu funksiyalarning skalyar ko’paytmasi

$$\left(y^{(k)},y^{(m)}\right)=\sum_{i=1}^{N-1}y_i^{(k)}y_i^{(m)}h=0,\ \ (k\ne m) \qquad (2.21)$$

bo’ladi.

3) $$\left\|y^{(k)}\right\|=\sqrt{\left(y^{(k)},y^{(k)}\right)}=\sqrt{\frac{l}{2}}. \qquad (2.22)$$

Demak,

$$X^{(k)}(x)=\sqrt{\frac{2}{l}}y^{(k)}(x) \qquad (2.23)$$

ko’rinishdagi funksiyalar (2.21) skalyar ko’paytma ma’nosida orthogonal hamda normalangan Sistema tashkil etadi, ya’ni $\left(x^{(k)},x^{(m)}\right)=\delta_{km}$, δ_{km} - Kronekker simvoli;

4) faraz qilaylik, $f(x)$ funksiya $\overline{\omega_n}$ to’rda berilgan bo’lib, to’rning chetki nuqtalarida
uning uchun quyidagi

$$f(0)=f(N)=0$$

chegaraviy shartlar bajarilsin, u holda $f(x)$ ni (2.15), (2.16) ning xos funksiyalari orqali quyidagi ko’rinishda yozishimiz mumkin:

$$f(x)=\sum_{k=1}^{N-1}f_kX^{(k)}(x), \qquad (2.24)$$

bu yerda

$$f_k=\left(f(x),\ X^{(k)}(x)\right) \qquad (2.25)$$

(2.25) ni e’tiborga olsak (2.24) dan

$$\| f\|^2 = \sum_{k=1}^{N-1} f_k^2 \qquad (2.26)$$

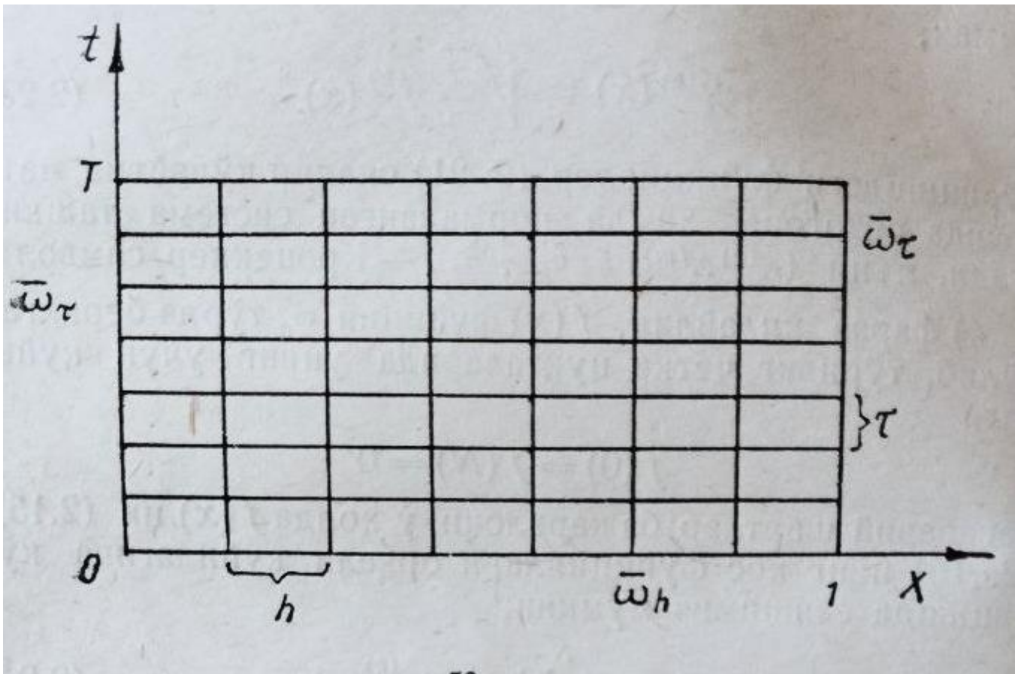
ekanligini ko'rish qiyin emas.

Ayirmali tenglamalar uchun xos son va xos funksiya tushunchalari shu tarzda kiritiladi. Endi bu tushunchalarni ayirmali tenglamalar sistemasini tekshirishga tatbiq qilish bilan shug'ullanamiz. Yana issiqlik o'tkazuvchanlik tenglamasiga qaytamiz:

$$\left.\begin{aligned} &\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + f, 0 < x < 1, 0 < t \le T, \\ &u(x,0) = u_0(x), 0 \le x \le 1, \\ &\left.\begin{aligned} &u(0,t) = u_1(t), \\ &u(1,t) = u_1(t), \end{aligned}\right\} 0 \le t \le T. \end{aligned}\right\} \qquad (2.27)$$

xOt tekislikda ($0 \le x \le 1,\ 0 \le t \le T$) h va τ qadam bilan to'r olaylik (50-shakl), bunda

$$\overline{\omega_{h,\tau}} = \left\{(ih, j\tau),\ i = \overline{0,N},\ j = \overline{0, j_0}\right\},$$



50- шакл.

$$h = \frac{1}{N},\ \ \tau = \frac{T}{j_0}.$$

Ichki tugunlar to'plamini

$$\omega_{h,\tau} = \left\{ \left(x_i, t_j \right),\ 1 \le i \le N-1,\ 1 \le j \le j_0 \right\}$$

orqali belgilaylik.

(2.27) differensialli masala uchun ayirmali masalani quyidagicha tuzamiz:

$$\left.\begin{array}{l} \dfrac{y_i^{j+1} - y_i^j}{\tau} = \Lambda\left(\sigma y_i^{j+1} + (1-\sigma) y_i^j \right) + \varphi_i^j,\ (x,t) \in \omega_{h,\tau} \\ y_0^j = u_1^i,\ \ y_N^j = u_2^j, \quad t \in \overline{\omega_\tau} \\ y_i^0 = y(x_i, 0) = u_0(x_i), \quad x \in \overline{\omega_h} \end{array}\right\}$$

(2.28)

Bu yerda φ_i^j tenglamaning o'ng tomoni $f(x,t)$ ni to'r nuqtalarida almashtiruvchi funksiya bo'lib,

$$\varphi_i^j = f\left(x_i, t_{\frac{j+1}{2}} \right), t_{\frac{j+1}{2}} = t_j + \frac{1}{2}\tau$$

(2.29)

va

$$\Lambda y_i = y_{\bar{x},x,i} = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$$

(2.30)

σ esa hozircha ixtiyoriy parametr.

(2.28) masala yechimini berilgan (2.27) masalaning aniq yechimiga yaqinlashish shartlarini aniqlash uchun $\overline{\omega}_{h,\tau}$ to'rda

$$z_i^j = y_i^j - u_i^j$$

(2.31)

funksiyani olib, uning normasini to'r qatlamlarida

$$\left.\begin{array}{l} \|z\| = \|z\|c = \max|z_i|, (0 \le i \le N), \\ \|z\| = \sqrt{\sum\limits_{i=1}^{N-1} z_i^2 h} \end{array}\right\} \quad (2.32)$$

ko'rinishlardan biri orqali ifodalaymiz. Agar

$$\left.\begin{array}{l} y_i^j = y, y_i^{j+1} = y, \\ y_i = \dfrac{1}{\tau}\left(y - y\right),\ \varphi_i^j = \varphi \end{array}\right\}$$

(2.33)

belgilar kiritsak, u holda (2. 28) o’rniga quyidagini yozish mumkin:

$$\left.\begin{array}{l} y_t = \Lambda\left(\sigma y + (1-\sigma)y\right) + \varphi, \quad (x,t)\in\omega_{h,\tau}, \\ y(0,t) = u_1(t),\ \ y(1,t) = u_2(t),\ \ t\in\overline{\omega_\tau}, \\ y(x,0) = u_0(x),\ \ x\in\overline{\omega_h} \end{array}\right\}$$

(2.34)

Bu sxemaning aniqligi haqidagi savolga javob berish uchun uning $y = y_i^j$ yechimi bilan (2.27) masalaning $u = u(x,\ t)$ yechimini taqqoslash kerak, chunki $u = u(x,\ t)$ (2.27) masalaning uzluksiz yechimidir. Buning uchun $u_i^i = u(x_i,\ t_j)$ deb $z_i^j = y_i^j - u_i^j$ ayirmani qaraymiz. Indekslarsiz belgilashga o’tib va $y = u + z$ ni (2.34) ga qo’yib yozsak (bu yerda u ni ma’lum funksiya deb olamiz),

$$\left.\begin{array}{l} z_t = \Lambda\left(\sigma \hat{z} + (1-\sigma)z\right) + \psi, \quad (x,t)\in\omega_{h,\tau}, \\ z(0,t) = z(1,t) = 0,\ \ t\in\omega_\tau, \\ z(x,0) = 0, \quad x\in\overline{\omega_h} \end{array}\right\} \qquad (2.35)$$

bo’ladi, bu yerda

$$\psi = \Lambda\left(\sigma u + (1-\sigma)u\right) - u_t + \varphi$$

(2.36)

(2.27) masalani (2.28) bilan taqribiy almashtirish jarayonida yo’l qo’yilgan xatolikdir.

Yangi belgilashlar kiritamiz:

$$u = \frac{\partial u}{\partial t},\ \ u' = \frac{\partial u}{\partial x}, \quad \bar{u} = u\left(x_i, t_{j+\frac{1}{2}}\right).$$

(2.37)

u funksiyani $\left(x_i, \bar{t} = t_{j+\frac{1}{2}}\right)$ nuqta atrofida Teyler qatoriga yoyib, kiritilgan belgilashlarni e’tiborga olsak,

$$\left.\begin{aligned} &u = \frac{u+u}{2} + \frac{u-u}{2} = \frac{u+u}{2} + \frac{\tau}{2}u_t, \\ &u = \frac{u+u}{2} - \frac{\tau}{2}u_t, \\ &\sigma u + (1-\sigma)u = \frac{u+u}{2} + (\sigma - 0,5)\tau u_t \end{aligned}\right\} \tag{2.38}$$

kelib chiqadi. (2.37) va (2.38) larga ko’ra ψ ni boshqacha yozish mumkin:

$$\psi = \frac{1}{2}\Lambda\left(u+u\right) + (\sigma - 0,5)\tau\Lambda u_t - u_t + \varphi. \tag{2.39}$$

Lekin

$$\left.\begin{aligned} &\Lambda u = u'' + \frac{h^2}{12}u^{\mathrm{IV}} + O\left(h^4\right) = Lu + \frac{h^2}{12}L^2u + O\left(h^4\right), \\ &\qquad\qquad L = \frac{\partial^2}{\partial x^2}. \\ &u = \bar{u} + \frac{1}{2}\tau\bar{\dot{u}} + \frac{\tau^2}{8}\bar{\ddot{u}} + O\left(\tau^3\right), \\ &u = \bar{u} - \frac{\tau}{2}\bar{\dot{u}} + \frac{\tau^2}{8}\bar{\ddot{u}} + O\left(\tau^3\right), \\ &\frac{u+u}{2} = \bar{u} + \frac{\tau^2}{8}\bar{\ddot{u}} + O\left(\tau^3\right), \\ &u_t = \frac{u-u}{\tau} = \bar{\dot{u}} + O\left(\tau^2\right) \end{aligned}\right\}$$

(2.40)

bo’lganligidan $\psi = \left(L\bar{u} - \bar{\dot{u}} + \varphi\right) + (\sigma - 0,5)\tau L\bar{\dot{u}} + \frac{h^2}{12}L^2\bar{u} + O\left(\tau^2 + h^4\right).$

Bundan tashqari, (2.27) ga asosan:

$$\left.\begin{aligned} &L\dot{u} = L^2u + Lf = u^{\mathrm{IV}} + f''. \\ &L^2u = L\dot{u} - Lf. \end{aligned}\right\} \tag{2.41}$$

Shuning uchun

$$\psi=\left(\varphi-\overline{f}\right)+\left[(\sigma-0,5)\tau+\frac{h^2}{12}\right]L\bar{u}-\frac{h^2}{12}Lf+O\left(\tau^2+h^4\right).$$

Hosil qilingan munosabat approksimatsiya haqida quyidagi natijalarga olib keladi:

1) agar $\sigma=0,5,\ \ \varphi=\overline{f}$ yoki $\varphi=\overline{f}+O\left(h^2+\tau^2\right),\ \ u\in C_3^4$ bo'lsa, taqribiy almashtirishdagi xatoning tartibi $O\left(h^2+\tau^2\right)$ ga teng. Bu yerda C_n^m orqali x o'zgaruvchisi bo'yicha m -tartibgacha va t o'zgaruvchisi bo'yicha n -tartibgacha uzluksiz hosilalari mavjud bo'lgan funksiyalar sinfi belgilangan;

2) agar $\sigma\neq 0,5,\ \ \varphi=\overline{f}+O\left(h^2+\tau^2\right),\ \ u\in C_2^4$ bo'lsa, taqribiy almashtirishdagi xatoning tartibi $O\left(h^2+\tau\right)$ dan ortmaydi;

3) agar $\sigma=\sigma_*=\frac{1}{2}-\frac{h^2}{12\tau}$

$$\varphi_i^j=\frac{5}{6}f_i^{j+\frac{1}{2}}+\frac{1}{12}\left(f_{i-1}^{j+\frac{1}{2}}+f_{i+1}^{j+\frac{1}{2}}\right)$$

bo'lsa (bu esa $\varphi=\overline{f}+\frac{h^2}{12}L\overline{f}$ ga teng kuchlidir), taqribiy almashtirishdagi xatoning tartibi $O\left(h^4+\tau^2\right)$ ga teng bo'ladi.

Endi (2.28) masalaning turg'unlik shartini tekshiraylik. Buning uchun bundan oldin Shturm-Luivill ayirmali masalasi uchun bayon qilingan xos funksiyalar va xos sonlar nazariyasidan foydalanamiz. Ma'lumki,

$$\left.\begin{aligned} &y=y+\tau y_t,\\ &\sigma y+\left(1-\sigma\right)y=y+\sigma\tau y_t.\end{aligned}\right\}\qquad(2.42)$$

Buni e'tiborga olgan holda (2.28) masalani bir jinsli chegaraviy shartlarda quyidagi ko'rinishda yozib olaylik.

$$\left.\begin{aligned} &y_t-\sigma\tau\Lambda y_t=\Lambda y+\varphi,\quad \left(x,\ t\right)\in\omega_{h\tau},\\ &y\left(0,t\right)=y\left(1,t\right)=0,\quad t\in\overline{\omega_\tau}\\ &y\left(x,0\right)=u_0\left(x\right),\quad x\in\overline{\omega_h}\end{aligned}\right\}\qquad(2.43)$$

Ayirmali sxema turg'un bo'lishi uchun

$$\left\|y(t)\right\|_{(1)} \le M_1 \left\|u_0\right\|_1 + M_2 \max_{0\le t'<t} \left\|\varphi(t')\right\|_{(2)}, \quad t\in\omega_\tau$$

(2.44)

shartning qanoatlantirilishi zarur, bu yerda M_1, M_2 lar h va τ ga bog'liq bo'lmagan musbat o'zgarmas sonlar bo'lib, $\|\cdot\|_{(1)}$ va $\|\cdot\|_{(2)}$ mos ravishda berilgan to'r funksiyalaridagi normalardir. Agar $f=0$ bo'lsa, ikkinchi qo'shiluvchi nolga teng va

$$\left\|y(t)\right\|_{(1)} \le M_1 \left\|u_0\right\|_{(1)}$$

(2.45)

munosabat boshlang'ich shart bo'yicha turg'unlikni ifodalaydi. Agar $u(x,0)=0$ bo'lsa,

$$\left\|y(t)\right\|_{(1)} \le M_2 \max_{0\le t'<t} \left\|\varphi(t')\right\|_{(2)}$$

(2.46)

munosabat tenglamaning o'ng tomoniga nisbatan turg'unlik shartini ifodalaydi. (2.43) masalaning yechimini $y=\bar{y}+y$ ko'rinishda yozaylik, bu yerda $\bar{y}$

$$\left.\begin{aligned} &y_t - \sigma\ \Lambda\ y_t = \Lambda y, \\ &y(0,t)=y(1,t)=0, \ \ y(x,0)=u_0(x), \end{aligned}\right\}$$

(2.47)

Bir jinsli masalaning yechimini, y esa bir jinsli bo'lmagan

$$\left.\begin{aligned} &y_t - \sigma\tau\Lambda\ y_t = \Lambda y + \varphi, \\ &y(0,\ t)=y(1,\ t)=0, \ \ \ y(x,\ 0)=0, \end{aligned}\right\}$$

(2.48)

masalaning yechimini ifodalaydi. (2.47) masalaning turg'unligini tekshirish uchun o'zgaruvchilarni ajratish usulidan (Fure usulidan) foydalanamiz va yechimni

$$y(x,t)=X(x)T(t)$$

(2.49)

ko'rinishda izlaymiz. (2.49) dan

$$\Lambda y = T\cdot\Lambda x, \ \ y_t = X\cdot T_t \qquad (2.50)$$

kelib chiqishini e'tiborga olgan holda, (2.47) dan

$$\frac{T-T}{\tau\left(\sigma T+(1-\sigma)T\right)}=\frac{\Lambda\lambda}{X}=-\lambda$$

(2.51)

ni hosil qilamiz, bu yerda

$T=T\left(t_{j+1}\right),\ \ T=T\left(t_j\right),\ \ \lambda-$ajratish parametri.

(2. 51) tenglikdan

$$T=qT$$

(2.52)

ni hosil qilamiz, bu yerda

$$q=\frac{1-(1-\sigma)\tau\lambda}{1+\tau\sigma\lambda}$$

(2.53)

$X(x)$ uchun (2.51) dan Shturm-Luivillning ayirmali tenglamasi kelib chiqadi:

$$\left.\begin{array}{l}\Lambda X(x)+\lambda X(x)=0, \quad (0<x=ih<1),\\ X(0)=X(1)=0.\end{array}\right\} \qquad (2.54)$$

Shunday qilib, (2.47) masala noldan farqli $y_{(k)}=T_k X^{(k)}\neq 0$ yechimga ega bo’lib, bu yerda T_k quyidagi tenglamadan aniqlanadi:

$$T_k=q_k T_k \text{ yoki } T_k^{j+1}=q_k T_k^j=\ldots=q_k^{j+1}T_k^0,$$

(2.55)

$$q_k=\frac{1-(1-\sigma)\tau\lambda_k}{1+\sigma\tau\lambda_k},$$

T_k^0-ixtiyoriy o’zgarmas son.

(2. 47) masalaning $y_{(k)}=T_k X^{(k)}$ ko’rinishdagi yechimini $k-$*nomerli garmonika* deyiladi.

Agar $|q_k|\leq 1$ bo’lsa, u holda j_0 ning o’sishi $(\tau\to 0)$ bilan, $\|y_k^j\|$ fiksirlangan $t=j\tau$ da o’smaydi:

$$\left\|y_{(k)}^{j+1}\right\|\leq\left\|y_{(k)}^{j}\right\|\leq\ldots..\leq\left\|y_{(k)}^{0}\right\|$$

va *garmonika turg’un* bo’ladi.

Agar hamma $\|q_k\| \le 1$ va $\left\|y_{(k)}^j\right\| \le \left\|y_{(k)}^0\right\|$ bo'lsa, u holda sxema har bir garmonikada turg'un deb ataladi.

(2.47) masalaning umumiy yechimi uning xususiy yechimlari yig'indisi ko'rinishida izlanadi(chunki masala chiziqli):

$$\hat{y} = \sum_{k=1}^{N-1} y_{(k)} = \sum_{k=1}^{N-1} T_k X^{(k)}$$

yoki

$$\hat{y} = \sum_{k=1}^{N-1} q_k T_k X^{(k)}.$$

So'nggi tenglikning har ikki tomonidan norma olsak;

$$\left\|\hat{y}\right\|^2 = \sum_{k=1}^{N-1} q_k^2 T_k^2 \left\|X^{(k)}\right\|^2 \le \max q_k^2 \sum_{k=1}^{N-1} T_k^2 = \max_k q_k^2 \|y\|^2$$

(2.56)

(bu yerda $X^{(k)}$ larning orthogonal ekanligini e'tiborga olsak).

$\sigma \ge \sigma_0$ bo'lganda, $\max_k |q_k| \le 1$ va (2.56) dan

$$\left\|\hat{y}\right\| \le \|y\|$$

yoki

$$\left\|y^{j+1}\right\| \le \left\|y^j\right\| \le \ldots.. \le \left\|y^0\right\| = \|u_0\|$$

kelib chiqqanligidan, (2.47) masalaning yechimi uchun quyidagi baho o'rinli bo'ladi:

$$\left\|y^j\right\| \le \|u_0\|, \qquad j = 1,\ 2,\ \ldots..$$

(2.57)

Shu bilan yechimning boshlang'ich shartga nisbatan turg'unligi isbotlandi.

Endi o'ng tomonga nisbatan turg'unlikni ko'raylik. Buning uchun (2.48) masalani qarab, uning yechimini ham

$$\hat{y} = \sum_{k=1}^{N-1} T_k X^{(k)}$$

(2.58)

ko’rinishda izlaymiz va φ ni $\left\{X^{(k)}\right\}$ funksiyalar bo’yicha qatorga yoyamiz:

$$\varphi=\sum_{k=1}^{N-1}\varphi_k X^{(k)}.$$

Bularni e’tiborga olsak, (2.48) dan [(2.37) ga q.]

$$\sum_{k=1}^{N-1}\left\{T_{kt}\left(1+\sigma\tau\lambda_k\right)+\lambda_k T_k-\varphi_k\right\}X^{(k)}=0.$$

(2.59)

$\left\{X^{(k)}\right\}$ larning ortogonalligi natijasida so’nggi tenglikdan

$$\tau\lambda_k T_k+\left(1+\sigma\tau\lambda_k\right)\left(T_k-T_k\right)=\tau\varphi_k$$

yoki

$$T_k=q_k T_k+\frac{\tau\varphi_k}{1+\sigma\tau\lambda_k}$$

(2.60)

ni hosil qilamiz, bu yerda

$$q_k=\frac{1-\left(1-\sigma\right)\tau\lambda_k}{1+\sigma\tau\lambda_k}.$$

Endi (2.60) ni (2.58) ga qo’ysak,

$$\hat{y}=\sum_{k=1}^{N-1}T_k X^{(k)}=\sum_{k=1}^{N-1}q_k T_k X^{(k)}+\tau\sum_{k=1}^{N-1}\frac{\varphi_k}{1+\sigma\tau\lambda_k}X^{(k)}$$

va bu tenglikning har ikki tomonlaridan norma olib, keyin uchburchak tengsizligidan $\left(\|v+w\|\right)\le\|v\|+\|w\|$ foydalansak,

$$\left\|\hat{y}\right\|\le\max_k\left|q_k\right|\left(\sum_{k=1}^{N-1}T_k^2\right)^{1/2}+\max_k\frac{\tau}{\left|1+\sigma\tau\lambda_k\right|}\left(\sum_{k=1}^{N-1}\varphi_k^2\right)^{\frac{1}{2}}$$

yoki

$$\left\|\hat{y}\right\|\le\max_k\left|q_k\right|\left\|y\right\|+\max_k\frac{\tau}{\left|1+\sigma\tau\lambda_k\right|}\left\|\varphi\right\|$$

(2.61)

kelib chiqadi. Faraz qilaylik, $\sigma \geq \sigma_0 > 0$ bo’lsin, u holda, $|q_k| \leq 1, \ \ 1+\sigma\tau\lambda_k \geq 1$ bo’ladi. Shuning uchun

$$\left\|\hat{y}\right\| \leq \|y\| + \tau\|\varphi\| \text{ yoki } \left\|y^{j'+1}\right\| \leq \left\|y^{j'}\right\| + \tau\left\|\varphi^{j'}\right\|.$$

So’nggi tengsizlikni $j' = 0, \ 1, \ 2, \ldots.., \ j$ bo’yicha yig’ib chiqamiz:

$$\left\|y^{j+1}\right\| \leq \sum_{j'=0}^{j} \tau\left\|\varphi^{j'}\right\|. \tag{2.62}$$

Faraz qilaylik, a) $\sigma \geq \sigma_4, \ \ \sigma_\sigma = \frac{1}{2} - \frac{1-\varepsilon}{4\tau}h^2, \ \ 0 < \varepsilon < 1$ va b) $\sigma < 0$ shartlar bajarilsin, u holda (2.61) dan

$$\left\|\hat{y}\right\| \leq \|y\| + \frac{1}{\varepsilon}\tau\|\varphi\|.$$

Buni $j' = 0, \ 1, \ \ldots.., \ j$ bo’yicha yig’ib chiqsak,

$$\left\|y^{j+1}\right\| \leq \frac{1}{\varepsilon}\sum_{j'=0}^{j} \tau\left\|\varphi^{j'}\right\|.$$

Shunday qilib, quyidagi teoremani isbot qildik.

Teorema. Agar $\sigma \geq \frac{1}{2} - \frac{h^2}{4\tau} = \sigma_0$ va $\sigma \geq 0$ *shartlar bajarilsa, (2.43) sxema boshlang’ich shart va o’ng tomon bo’yicha turg’un bo’lib, (2.43) ning yechimi uchun quyidagi baho o’rinli bo’ladi:*

$$\left\|y^{j+1}\right\| \leq \|u_0\| + \sum_{j'=0}^{j} \tau\left\|\varphi^{j'}\right\|.$$

Agar $\sigma < 0$ bo’lsa, (2.43) sxemaning turg’un bo’lishi uchun

$$\sigma \geq \sigma_\varepsilon, \ \ \sigma_\varepsilon = \frac{1}{2} - \frac{(1-\varepsilon)h^2}{4\tau}$$

$(0 < \varepsilon < 1)$ shartning bajarilishi yetarlidir ($\varepsilon \in (0,1)$ ixtiyoriy o’zgarmas son bo’lib, h va τ ga bog’liq emas). Bu holda (2.43) ning yechimi uchun quyidagi baho o’rinlidir:

$$\left\|y^{j+1}\right\| \leq \|u_0\| + \frac{1}{\varepsilon}\sum_{j'=0}^{j} \tau\left\|\varphi^{j'}\right\|.$$

Demak, *xos sonlar va xos funksiyalar yordamida diferensial tenglamalar kabi ayirmali tenglamalar uchun ham yechim qurish va bu yechimning normasini tekshirish orqali approksimatsiya, turg'unlik va yaqinlashish masalalarini hal qilish mumkin.*

Shunday qilib, biz oddiy issiqlik o'tkazuvchanlik tenglamasi uchun ayirmali sxemalarni o'rganib chiqdik. Shuni eslatib o'tish kerakki, murakkab differensial tenglamalarni katta aniqlik bilan almashtiradigan ayirmali sxemalarni o'rganish zaruriyati tug'ilganida asosiy masalalardan biri hamma vaqt sxemalarning turg'un ekanligini tekshirishdir, chunki shunday sxemalargina amaliy nuqtai nazardan katta ahamiyatga ega. Biz kelgusida ayirmali sxemalarning turg'unlik masalasini tekshirishning quyidagi usullarini ham ishlatamiz:

1) maksimum prinsipi asosida sxemalarning turg'unligini tekshirish;
2) ayirmali sxemalar indeksi yordamida sxemalarning turg'unligini tekshirish;
3) birlik xatoning o'sishini o'rganish hisobiga turg'unlikni tekshirish;
4) o'zgaruvchilarni ajratish usuli asosida sxemalarning turg'unligini tekshirish; yuqorida biz xuddi shu usuldan foydalandik.

Bu usullarning hammasi asosan o'zgarmas koeffitsientli differensial tenglamalarga mos kelgan ayirmali tenglamalarning turg'unligini tekshirish uchun yaraydi.

O'zgaruvchi koeffitsientli differensial tenglamalarga mos keladigan ayirmali tenglamalarning turg'unligini tekshirish odatda juda murakkab masalalardan biri hisoblanadi. Amaliyotda uzluksiz koeffitsientli tenglamalarga mos keladigan ayirmali tenglamalarning turg'unligini quyidagi prinsip asosida tekshiriladi.

Tenglamadagi barcha o'zgaruvchi koeffitsientlarni soxaning qandaydir P nuqtasidagi qiymati bilan almashtiriladi. Agar ixtiyoriy P uchun hosil bo'lgan ayirmali masala turg'un bo'lsa, u holda o'zgaruvchi koeffitsientli ayirmali masala turg'un deb hisoblanadi.

Chiziqli bo'lmagan differensial tenglamalarni yechish uchun mos keladigan ayirmali sxemalarning turg'unligini tekshirish yana ham og'irroq masaladir. Bu holda sxemaning turg'unligini chiziqli holga keltirilgan tenglamaning yechimi atrofida tekshiriladi. Bunday sxemalarni biz mexanika masalalarini qarayotganimizda keltiramiz.

Endi biz bundan keying paragraflarda to'r usulini umumiyroq ko'rinishda bayon qilamiz, yaqinlashish va turg'unlikning o'zaro aloqasini ko'rsatamiz hamda sxemalarni yuqorida sanab o'tilgan usullarning ba'zilari asosida tekshirishga to'xtalamiz.

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